

TAUP-2412-97  
February 1, 2008

# Screening and confinement in large $N_f$ $QCD_2$ and in $N = 1$ $SYM_2$

A. Armoni and J. Sonnenschein <sup>1</sup>

*School of Physics and Astronomy  
Beverly and Raymond Sackler Faculty of Exact Sciences  
Tel Aviv University, Ramat Aviv, 69978, Israel*

## Abstract

The screening nature of the potential between external quarks in massless  $SU(N_c)$   $QCD_2$  is derived using an expansion in  $N_f$ - the number of flavors. Applying the same method to the massive model, we find a confining potential. We consider the  $N = 1$  super Yang Mills theory, reveal certain problematic aspects of its bosonized version and show the associated screening behavior by applying a point splitting method to the scalar current.

---

<sup>1</sup>Work supported in part by the Israel Science Foundation, and the US-Israel Binational Science Foundation

# 1 Introduction

Large  $N_c$  expansion[1], and strong coupling expansion [2] were shown to be powerful tools in analyzing various aspects of  $QCD_2$ . In the present work we demonstrate the usefulness of yet another technique, that of expanding in large number of flavors  $N_f$ . We apply this method to derive further evidence for the screening behavior between external quarks in the massless theory, and the confining one in the massive theory[3].

The screening nature of the potential was argued in ref.[3] by substituting a static (abelian) solution of the equations of motion for the gauge configuration into the expression of the potential. This approach falls short in proving the screening nature of the interaction between the external quarks because of the following drawbacks. (i) Classical configurations cannot always recast for the quantum behavior of a system. (ii) Treating the external quarks as classical sources is strictly speaking justified only for large color representations in a similar way that only particles with large angular momentum can be described in quantum mechanics as c-number quantities.

In the present paper we suggest improvements that enable one to overcome these obstacles. To justify the fact that functional integral is dominated by the saddle-point configurations we introduce  $N_f$  flavor degrees of freedom and analyze the system in the limit of large  $N_f$ . It is easy to realize, especially in the bosonization formulation, that the action can be brought into a form where it is multiplied by an overall factor of  $N_f$ . This obviously implies that  $\frac{1}{N_f}$  plays the role of  $\hbar$  so that the limit of  $N_f \rightarrow \infty$  corresponds to the classical limit of  $\hbar \rightarrow 0$ .

Moreover, the abelian gauge configuration that solves the equations of motion and is used in [3] to derive a screening potential, can be used as a first iteration in a systematic expansion of the gauge configuration in powers of  $\frac{1}{N_f}$ . One can thus check whether the screening nature persists also in corrections which are proportional to higher powers of  $\frac{1}{N_f}$ . However, if one presents the external quarks in terms of commuting functions as was used in [3], then the iterated gauge field solution at any order will remain abelian if the leading one is. By abelian we mean that commutator terms in the equations of motions vanish. One may be suspicious that in that way one abelianize  $QCD_2$  in an unjustified manner and hence the form of the potential cannot be faithfully examined. This situation can be avoided either by searching

genuine non-abelian solutions of the equations of motions [4] or by using a different method of representing the external quarks so that the non-abelian commutator terms do not vanish. A proposal of this nature was introduced in refs.[5, 6] where the density of the external quarks was represented in terms of non-commuting matrices. We compute the potential including corrections of next and next to next order corrections and show that it tends to a constant for large separation distance, thus it exhibits a screening behavior.

Two dimensional massless  $QCD$  admits a universal nature[7]. The introduction of mass terms to the quarks creates a much more dramatic alteration in the system than in the real world 4D  $QCD$ . This property manifests itself in the baryonic spectrum[2] as well as in the form of the potential between external quarks. External quarks in the fundamental representations that were screened by massless adjoint dynamical fermions, find themselves in the confining phase once the dynamical fermions acquire mass. In ref.[3] this phenomenon was derived for the analog abelian case as well as for  $QCD_2$  with two and three colors. In [4] it was further shown for a general  $SU(N_c)$  gauge group by bosonizing both the dynamical and the external quarks and the conditions for finite energy quark configurations were analyzed. An explicit determination of the potential, in a similar manner to way it was done in the massless case, is technically much more tedious due to the form of the bosonized mass term. Once again the expansion in large  $N_f$  comes to our rescue. The mass term can be expanded in terms of a (non-local) power series of the color current divided by  $N_f$ . Restricting the expansion to the lowest power of  $\frac{1}{N_f}$  renders the equations of motion in terms of the currents to tractable ones. Using this method we show that the leading solution for the potential exhibits a confining behavior.

A natural framework that “embeds” the  $QCD_2$  model with adjoint fermions is that of  $N = 1$  super Yang-Mills theory. In ref. [3] the potential of that model is conjectured to be a screening one. We show that a naive bosonization of the fermionic degrees of freedom of the model leads to a wrong picture. It is further shown that by considering the contribution of the scalar field to the one loop vacuum polarization one indeed discover the non-confining potential.

The paper is organized as follows. In section 2 the equations of motion of multi-flavor massless  $QCD_2$  in the presence of external quarks are derived in the  $A_- = 0$  gauge. A large  $N_f$  iteration analysis is described in section

3. The potential is shown to be a screening one even for a non-commuting presentation of the external density. Section 4 is devoted to the massive multi-flavor case. Expressing the mass term in a power series in the currents we derive the confining potential for that model. An analysis of the potential for the  $N = 1$  supersymmetric Yang-Mills theory is presented in section 5. We show that a one loop effect in the scalar sector which is the analog of the anomaly term for the fermions combines with the latter to produce a screening potential.

## 2 Multi-flavor $QCD_2$ with external source

Massless multi-flavor  $QCD_2$  with fermions in the fundamental representation of  $SU(N_c)$  and external current coupled to the gauge field is described by the following action

$$S = \int d^2x \operatorname{tr} \left( -\frac{1}{4} F_{\mu\nu}^2 + i\bar{\Psi} \not{D} \Psi - e A_\mu j_{ext}^\mu \right) \quad (1)$$

where  $\Psi = \Psi^{i\alpha}$ ,  $i = 1 \dots N_c$  and  $\alpha = 1 \dots N_f$  and the trace is over both the color and flavor indices. Variation with respect to the Dirac field and gauge fields leads to the following equations of motion:

$$D_\mu J^\mu = 0 \quad (2)$$

$$D_\mu j_{ext}^\mu = 0 \quad (3)$$

$$D_\mu F^{\mu\nu} = e(J^\nu + j_{ext}^\nu) \quad (4)$$

where  $D_\mu = \partial_\mu + e[A_\mu, \ ]$ . The first (2) and the second (3) equations are the covariant conservation of the dynamical and external vector currents, respectively. In addition the chiral anomaly equation reads

$$D_\mu J^{5\mu} = \frac{eN_f}{2\pi} \epsilon^{\mu\nu} F_{\mu\nu} \quad (5)$$

where  $J^{5\mu} = \bar{\Psi} \gamma^5 \gamma^\mu \Psi$ . The anomaly equation can be obtained by the equations of motion of the bosonized action of (1) (see for instance[2]).

In order to maintain gauge invariance of the action (1), the external current should be covariantly conserved. This implies that, in contrast to the abelian case, one cannot set  $j_{ext}^0$  as a time independent function and

then fix  $j_{ext}^1$  to be zero. Instead we fix the value of  $j_{ext}^0$  and treat  $j_{ext}^1$  as a “dynamical” variable of the problem.

By choosing the  $A_- = 0$  gauge, using light cone coordinates  $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$ , and the two dimensional property  $J^{5\mu} = -\epsilon^{\mu\nu} J_\nu$ , we translate equations (2),(5) and (4) into

$$\begin{aligned} \partial_+ J^+ + \partial_- J^- + e[A_+, J^+] &= 0 \\ \partial_+ J^+ - \partial_- J^- + e[A_+, J^+] &= \frac{eN_f}{\pi} \partial_- A_+ \\ -\partial_-^2 A_+ &= e(J^+ + j_{ext}^+) \end{aligned} \tag{6}$$

In our analysis we will be interested in static solutions of the equations that correspond to static external sources. For this case the set of equations combined with the external source constraint takes the following simplified form

$$\begin{aligned} \frac{1}{\sqrt{2}} \partial_1 J^+ + e[A_+, J^+] &= -\frac{eN_f}{2\pi} \frac{1}{\sqrt{2}} \partial_1 A_+ \\ \frac{1}{2} \partial_1^2 A_+ &= -e(J^+ + \frac{1}{\sqrt{2}} \rho + \frac{1}{\sqrt{2}} j) \\ \partial_1 j + \frac{1}{\sqrt{2}} e[A_+, \rho] + \frac{1}{\sqrt{2}} e[A_+, j] &= 0 \end{aligned} \tag{7}$$

where we have used the notation  $j \equiv j_{ext}^1$  and  $\rho \equiv j_{ext}^0$ . Given  $\rho$ , equations (7) can be used to determine the dynamical variables  $A_+$ ,  $J^+$  and  $j$  once boundary conditions are specified.

The gauge field  $A_+$  itself, is not the physical quantity of interest, however, once it is determined, the potential energy between the external charges can be found by substituting it in the effective action. Using the equations of motion the potential takes the following form:

$$V = \frac{1}{2} e \int dx \text{tr}(A_+ j_{ext}^+) = \frac{1}{2} e \int dx A_+^a \frac{1}{\sqrt{2}} (\rho + j)^a \tag{8}$$

where  $a$  are the adjoint indices of the color group  $a = 1, \dots, N_c^2 - 1$ . As was discussed in the introduction, the behavior of the potential as a function of the distance between the external quark and the anti-quark determines whether the theory is screening or confining. A linear potential means a

constant force, namely, confinement, while a constant potential (at large distances) means screening.

We are now facing the question of how to incorporate the external sources. Consider a system of a quark in the fundamental representation placed at a distance of  $2R$  from an anti-quark that transforms in the anti-fundamental representation. This can be expressed as the following classical c-number function

$$\rho^a = \delta^{a1}(\delta(x - R) - \delta(x + R)) \quad (9)$$

Strictly speaking, one is allowed to introduce classical charges and neglect quantum fluctuations only if the external charges transform in a large color representation. (This is an analog of the statement that only for quantities of large angular momentum quantum fluctuations are suppressed.) Moreover, by choosing (9), there is an obvious “abelian” self-consistent solution[3] of the equations (7) for which all the dynamical quantities ( $A_+$ ,  $J^+$  and  $j$ ) points in the ‘1’ direction and thus all the commutators vanish. This solution does not reflect the non-abelian nature of the theory. This situation is not avoided also when one uses an iterative expansion in large  $N_f$ , as is described in the next section. Hence, it seems that deriving conclusions from the abelian solution based on the use of (9) is unjustified and may miss the true nature of the interaction. One way to overcome these obstacles is to search for “truly” non-abelian solutions of the equations. This approach was followed in ref. [4]. Here we proceed by implementing Adler’s[5] semi-classical approach for introducing static external quark charges. In this approach the quarks color charges satisfy non-abelian  $SU(N_c)$  color algebra so that the external quark charge density takes the form

$$\rho^a = Q^a \delta(x - R) + \bar{Q}^a \delta(x + R) \quad (10)$$

$Q^a$  and  $\bar{Q}^a$  are in  $(\mathbf{N}_c, 1)$  and  $(1, \bar{\mathbf{N}}_c)$  representations of  $SU(N_c) \otimes SU(N_c)$  group respectively. The algebra of those operators which was worked out in [5, 6] is reviewed briefly in the appendix. Note that unlike what followed from the classical expression (9), now since  $Q^a$  and  $\bar{Q}^a$  do not commute, “non-abelian” solutions are expected. In the following section we apply a systematic  $\frac{1}{N_f}$  expansion of  $A_+$  and derive such a solution.

### 3 Large $N_f$ expansion and non-Abelian solutions

The form of the set of equations (7) suggests a natural  $\frac{1}{N_f}$  expansion. Large  $N_f$ , with  $\frac{e^2 N_f}{\pi} \equiv \mu^2$  fixed, means weak coupling constant  $e$ . In this case a procedure of extracting a solution by iterations can be applied in the following way. A solution for  $A_+$ ,  $J^+$  and  $j$  of the equations expanded to a given order in  $e$  is inserted back to (7) as a source to determine the next order solution. A similar treatment in four dimensions is given in refs.[8, 9].

The formal expansion in  $e$  is as follows

$$\begin{aligned} A_+ &= eA^{(1)} + e^3 A^{(3)} + e^5 A^{(5)} + \dots \\ J^+ &= J^{(0)} + e^2 J^{(2)} + e^4 J^{(4)} + \dots \\ j &= e^2 j^{(2)} + e^4 j^{(4)} + e^6 j^{(6)} + \dots \end{aligned} \tag{11}$$

Substituting  $A_+$ ,  $J^+$  and  $j$  back in (7) one gets equations for  $A^{(i)}$ ,  $J^{(i-1)}$  and  $j^{(i+1)}$  which to lowest order in  $\frac{1}{N_f}$  take the form

$$\begin{aligned} \frac{1}{\sqrt{2}} \partial_1 J^{(0)} &= -\frac{1}{2} \mu^2 \frac{1}{\sqrt{2}} \partial_1 A^{(1)} \\ \frac{1}{2} \partial_1^2 A^{(1)} &= -(J^{(0)} + \frac{1}{\sqrt{2}} \rho) \end{aligned}$$

Assuming vanishing currents at infinity the equations can be rewritten as

$$\begin{aligned} J^{(0)} &= -\frac{1}{2} \mu^2 A^{(1)} \\ (\partial_1^2 - \mu^2) A^{(1)} &= -\sqrt{2} \rho \end{aligned}$$

The solution for  $A^{(1)}$  is

$$A^{(1)} = \frac{\sqrt{2}}{2\mu} (Q e^{-\mu|x-R|} + \bar{Q} e^{-\mu|x+R|}) \tag{12}$$

This is the “abelian” solution of [3] with a replacement of the c-number charges with the non-commuting ones. Whereas in the case of  $QED_2$  coupled to external sources, this is (with c-number charges) an *exact* solution, in the

present case it is the leading (in  $\frac{1}{N_f}$ ) contribution to  $A_+$ . The next to leading order set of equations is

$$\begin{aligned}\frac{1}{\sqrt{2}}\partial_1 J^{(2)} + [A^{(1)}, J^{(0)}] &= -\frac{1}{2}\mu^2 \frac{1}{\sqrt{2}}\partial_1 A^{(3)} \\ \frac{1}{2}\partial_1^2 A^{(3)} &= -(J^{(2)} + \frac{1}{\sqrt{2}}j^{(2)}) \\ \partial_1 j^{(2)} + \frac{1}{\sqrt{2}}[A^{(1)}, \rho] &= 0\end{aligned}$$

Substituting  $A^{(1)}$  and  $J^{(0)}$  we find

$$\begin{aligned}J^{(2)} &= -\frac{1}{2}\mu^2 A^{(3)} \\ (\partial_1^2 - \mu^2)A^{(3)} &= -\sqrt{2}j^{(2)}\end{aligned}$$

with  $j^{(2)}$ , which is determined by the previous iteration, of the form

$$j^{(2)} = -\frac{1}{\sqrt{2}}\frac{1}{\partial_1}[A^{(1)}, \rho] = \frac{1}{4\mu}[\bar{Q}, Q]e^{-2\mu R}(\epsilon(x+R) - \epsilon(x-R)) \quad (13)$$

where  $\epsilon(x)$  is the step function  $\epsilon(x) = 1$  for  $x > 0$  and  $\epsilon(x) = -1$  otherwise and  $\frac{1}{\partial_1}$  denotes the integral  $\int_{-\infty}^x dx'$ .

The latter expression acts as a source to  $A^{(3)}$  which leads

$$A^{(3)} = \frac{\sqrt{2}}{4\mu^3}[\bar{Q}, Q]e^{-2\mu R}(\epsilon(x+R)(1 - e^{-\mu|x+R|}) - \epsilon(x-R)(1 - e^{-\mu|x-R|})) \quad (14)$$

Far from the sources  $A^{(3)}$  approaches zero. This indicates that the potential is approaching a constant value when one quark is taken to be far from the other. However, as we shall see,  $A^{(5)}$  will be needed to observe the first correction to the potential. This calculation is written in the Appendix.

We substitute now the expression found for  $A_+$  and  $j^+$  in (8) to derive the following potential

$$\begin{aligned}V &= \frac{1}{2}e \int dx A_+ \frac{1}{\sqrt{2}}(\rho + j) \\ &= \frac{e}{2\sqrt{2}} \int dx (eA^{(1)} + e^3 A^{(3)} + e^5 A^{(5)} + \dots)(\rho + e^2 j^{(2)} + e^4 j^{(4)} + \dots) \\ &= \frac{1}{2\sqrt{2}} \int dx (e^2 A^{(1)}\rho + e^4 (A^{(1)}j^{(2)} + A^{(3)}\rho) + e^6 (A^{(1)}j^{(4)} + A^{(3)}j^{(2)} + A^{(5)}\rho) + \dots)\end{aligned}$$



All the  $e^4$  terms vanish because they contain  $[Q, \bar{Q}]Q \sim f^{abc}Q^a\bar{Q}^bQ^c$  part which vanishes. Therefore the first non-trivial correction is  $O(e^6)$ .

$$V(2R) = \frac{e^2}{4\mu}(QQ + \bar{Q}\bar{Q} + 2Q\bar{Q}e^{-2\mu R}) \quad (15)$$

$$+ \frac{e^6}{8\mu^5}[\bar{Q}, Q]^2 \left[ (1 - e^{-2\mu R})^2 - 2\mu R e^{-2\mu R}(1 - e^{-2\mu R}) \right]$$

Let us compute the group constants that appear in (15).  $QQ$  stands for  $Q^aQ^a$  which is the second Casimir operator of the fundamental representation-  $Q^aQ^a = \frac{N_c^2-1}{2N_c}$ . Similarly  $\bar{Q}\bar{Q} = \frac{N_c^2-1}{2N_c}$ . The value of  $Q\bar{Q}$  is determined easily by using  $2Q^a\bar{Q}^a = (Q^a + \bar{Q}^a)(Q^a + \bar{Q}^a) - Q^aQ^a - \bar{Q}^a\bar{Q}^a$ . In the singlet coupled state  $Q^a + \bar{Q}^a = 0$  and hence  $Q^a\bar{Q}^a = -\frac{N_c^2-1}{2N_c}$ .

The value of  $[Q, \bar{Q}]^2$  is computed by the  $Q\bar{Q}$  algebra (see the Appendix), it is  $[Q, \bar{Q}]^2 = \frac{N_c^2}{2} \frac{N_c^2-1}{2N_c}$ .

Defining  $d \equiv 2R$ , the potential takes the form:

$$V(d) = \mu \frac{\pi}{2N_f} \frac{N_c^2-1}{2N_c} (1 - e^{-\mu d}) \quad (16)$$

$$+ \mu \left( \frac{\pi}{2N_f} \right)^3 \frac{N_c^2}{2} \frac{N_c^2-1}{2N_c} \left( (1 - e^{-\mu d})^2 - \mu d e^{-\mu d} (1 - e^{-\mu d}) \right)$$

Thus, the potential that includes the first correction to the abelian one approaches a constant value at large distances where the force between the external quark and the anti-quark vanishes. The obvious question now is whether we can infer from this result that the screening nature of the interaction remains valid to all orders in  $\frac{1}{N_f}$ . We cannot provide a general proof of that statement, but we believe that indeed that is the exact nature of the interaction. This is based on the following argument. The structure of all higher contributions is  $(\partial_1^2 - \mu^2)A = -j$ , where  $j$  is determined by previous iterations. Therefore,  $j$  would always vanish far from the sources and consequently the gauge field  $A$  would exhibit the same behavior.

## 4 Large $N_f$ expansion of massive $QCD_2$

Let us consider now the case of massive dynamical quarks. Whereas, for the massless case the equations that determine the potential can be derived

using both the fermionic picture and the bosonized one, here we can apply our analysis only in the bosonization description. The bosonized action of massive  $QCD_2$  with  $N_f$  fundamental representations in the gauge  $A_- = 0$  takes the following form[2]

$$S = S_{(N_c)}^{WZW}(g) + S_{(N_f)}^{WZW}(h) + \frac{1}{2} \int d^2x \partial_\mu \phi \partial^\mu \phi + S_{(1)}^{WZW}(l) + \int d^2x \text{tr} \left[ m^2 : (ghl \exp(-i\sqrt{\frac{4\pi}{N_c N_f}} \phi) + \exp(i\sqrt{\frac{4\pi}{N_c N_f}} \phi) l^\dagger h^\dagger g^\dagger) :_{\tilde{m}} + \frac{1}{2} (\partial_- A_+)^2 - e A_+ (J^+ + j_{ext}^+) \right],$$

where  $\text{tr}$  is over  $U(N_f \times N_c)$ ,  $g \in SU(N_f)$ ,  $h \in SU(N_c)$ ,  $\exp(-i\sqrt{\frac{4\pi}{N_c N_f}} \phi) \in U(1)$ ,  $l \in U(N_c \times N_f)/SU(N_c) \times SU(N_f) \times U(1)$ ,  $S_{(k)}^{WZW}$  is the level  $k$  WZW action.  $:_{\tilde{m}}$  denoted normal ordering at mass scale  $\tilde{m}$  and  $m^2 = m_q \tilde{m} C$  where  $m_q$  is the quark mass, and  $C = \frac{1}{2} e^\gamma$  with  $\gamma$  Euler's constant.

The bosonized mass term is the only term in the action that couples the colored and flavored sectors. Since we are interested only in the form of the potential we can simplify the analysis of the equations of motion of the full theory by restricting ourself only to the colored sector. This can be achieved by setting  $g = 1$ ,  $l = 1$  and  $\phi = 0$ . The normal ordering can be performed first at the scale  $\mu = \frac{e\sqrt{N_f}}{\sqrt{\pi}}$  and then one can replace the two mass scales  $\mu$  and  $m_q$  by a single scale by normal ordering at a certain scale  $m$ [2]. In that case  $m = [N_f m_q C \mu^{1-\Delta_c}]^{\frac{1}{2-\Delta_c}}$  where  $\Delta_c$  the dimension of  $h$  is given by  $\Delta_c = \frac{N_c^2 - 1}{N_c(N_c + N_f)}$

which leads the following action

$$S = S_{(N_f)}^{WZW}(h) + \int d^2x \text{tr} \left[ m^2 (h + h^{-1}) + \frac{1}{2} (\partial_- A_+)^2 - e A_+ (J^+ + j_{ext}^+) \right], \quad (17)$$

and the trace is over the color degrees of freedom. Equations (6) now read

$$\partial_+ J^+ + e[A_+, J^+] = \frac{e N_f}{2\pi} \partial_- A_+ - i m^2 (h^{-1} - h) \quad (18)$$

$$-\partial_-^2 A_+ = e(J^+ + j_{ext}^+) \quad (19)$$

The difference from the equations derived in the massless case is that here there is a dependence on  $h$  on top of the dependence on the currents. Unlike the abelian theory here we can write a closed expression for the group element ( $h$ ) only in a formal form in terms of the current. However we can express it as an expansion in powers of  $J^+$ . Recall tht the current  $J^+$  is related to the color group element  $h$  by

$$J^+ = \frac{iN_f}{2\pi} h \partial_- h^{-1} \quad (20)$$

The expanding of the inverse relation takes the form

$$h = 1 - \frac{2\pi}{iN_f} \frac{1}{\partial_-} J^+ + \left(\frac{2\pi}{iN_f}\right)^2 \left( \left(\frac{1}{\partial_-} J^+\right)^2 - \frac{1}{\partial_-} \left(\frac{1}{\partial_-} J^+\right) J^+ \right) + \dots \quad (21)$$

$$h^{-1} = 1 + \frac{2\pi}{iN_f} \frac{1}{\partial_-} J^+ + \left(\frac{2\pi}{iN_f}\right)^2 \frac{1}{\partial_-} \left( \left(\frac{1}{\partial_-} J^+\right) J^+ \right) + \dots \quad (22)$$

where the dots denote additional terms which are higher powers of  $J^+$ . This expansion makes sense upon the substitution of  $J^+ = \sum_{n=0}^{\infty} \left(\frac{1}{N_f}\right)^n J^{(2n)}$  given in eqn.(11). Moreover, note that even for the free level  $N_f$  WZW model this expansion is justified since  $J^+$  behaves like  $J^+ \sim \sqrt{N_f}$  as can be deduced from the associated affine Lie algebra. For the expression needed in (18) we get

$$h^{-1} - h = 2 \frac{2\pi}{iN_f} \frac{1}{\partial_-} J^+ + \left(\frac{2\pi}{iN_f}\right)^2 \frac{1}{\partial_-} \left[ \frac{1}{\partial_-} J^+, J^+ \right] + \dots, \quad (23)$$

Thus, the set of equations (7), for static solutions, take the following form in the presence of mass

$$\begin{aligned} \frac{1}{\sqrt{2}} \partial_1 J^+ + e[A_+, J^+] &\simeq -\frac{eN_f}{2\pi} \frac{1}{\sqrt{2}} \partial_1 A_+ \\ &- im^2 \left( -2 \frac{\sqrt{2} 2\pi}{iN_f} \frac{1}{\partial_1} J^+ + \left(\frac{\sqrt{2} 2\pi}{iN_f}\right)^2 \frac{1}{\partial_1} \left[ \frac{1}{\partial_1} J^+, J^+ \right] \right) \\ \frac{1}{2} \partial_1^2 A_+ &= -e \left( J^+ + \frac{1}{\sqrt{2}} \rho + \frac{1}{\sqrt{2}} j \right) \\ \partial_1 j + \frac{1}{\sqrt{2}} e[A_+, \rho] + \frac{1}{\sqrt{2}} e[A_+, j] &= 0 \end{aligned}$$

which to leading order, in  $\frac{1}{N_f}$ , is

$$\begin{aligned}\frac{1}{\sqrt{2}}\partial_1 J^+ &= -\frac{eN_f}{2\pi}\frac{1}{\sqrt{2}}\partial_1 A_+ + im^2 2\frac{\sqrt{2}2\pi}{iN_f}\frac{1}{\partial_1}J^+ \\ \frac{1}{2}\partial_1^2 A_+ &= -e(J^+ + \frac{1}{\sqrt{2}}\rho)\end{aligned}$$

Eliminating  $J^+$ , we find the following equation for  $A_+$

$$(1 + \frac{e^2 N_f^2}{8\pi^2 m^2})\partial_1^2 A_+ \simeq -e\sqrt{2}\rho \quad (24)$$

The solution of the equation in the presence of (10) is

$$A_+ = -\frac{e}{\sqrt{2}}(1 + \frac{e^2 N_f^2}{8\pi^2 m^2})^{-1}(Q | x - R | + \bar{Q} | x + R |) \quad (25)$$

Substituting  $A_+$  in the potential yields,

$$V = -\frac{e^2}{2}(1 + \frac{e^2 N_f^2}{8\pi^2 m^2})^{-1}Q\bar{Q} \times 2R \quad (26)$$

Using the definition  $d \equiv 2R$ , substituting  $m^2$ ,  $\mu^2$  and  $Q\bar{Q}$  which is minus the quadratic casimir operator of the test charges in a general representation  $\mathcal{R}$  we obtain

$$V = \frac{\mu^2 \pi}{2N_f}(1 + \frac{\mu}{8\pi C m_q})^{-1}C_2(\mathcal{R}) \times d \quad (27)$$

The same expression for the potential in the abelian case was obtained in [3]. Thus the dominant  $\frac{1}{N_f}$  contribution exhibits a confinement behavior. It should be emphasized that in the above analysis it is assumed that the external charges cannot be composed by the dynamical ones. This is the analog of the abelian case where the external charges are not integer multiple of the dynamical charges[10]. It is well known that in systems, where the latter does not hold, the string between the external quark and antiquark can be torn apart by a pair creation. Similarly in the non-abelian case we expect that when the test charges can be composed of a multiplication of the dynamical charges the string could be torn. Confinement is restored (and (27) is valid) only when when the test charges *cannot be composed* of the dynamical charges[11]. It is clear that in the latter case the confining nature of the theory survives higher order  $\frac{1}{N_f}$  corrections.

## 5 Supersymmetric Yang-Mills

Next, we would like to investigate the large distance behavior of  $N = 1$  two dimensional supersymmetric Yang-Mills theory, and test the conjecture that the system is in a screening phase [3].

The  $SYM_2$  action [12] is the following

$$S = \int d^2x \operatorname{tr} \left( -\frac{1}{4} F_{\mu\nu}^2 + i\bar{\lambda} \not{D} \lambda + \frac{1}{2} (D_\mu \phi)^2 + 2ie\phi \bar{\lambda} \gamma_5 \lambda \right), \quad (28)$$

where  $A_\mu$  the gluon field,  $\lambda$  the gluino and  $\phi$  a pseudo-scalar - the components of the vector supermultiplet- transform in the adjoint representation of  $SU(N_c)$ .

The action is invariant under the following supersymmetric transformations

$$\begin{aligned} \delta A_\mu &= i\bar{\epsilon} \gamma_5 \gamma_\mu \sqrt{2} \lambda \\ \delta \phi &= -\bar{\epsilon} \sqrt{2} \lambda \\ \delta \lambda &= -\frac{1}{2\sqrt{2}} \epsilon \epsilon^{\mu\nu} F_{\mu\nu} + \frac{i}{\sqrt{2}} \gamma^\mu \epsilon D_\mu \phi \end{aligned}$$

where  $\epsilon$  is the fermionic parameter of transformation. In order to determine the behavior of the theory it seems natural to follow the same procedure used in the previous sections, namely, to bosonize the fermionic degrees of freedom. Then, to proceed by solving the equations of motion for the gauge fields and deducing the potential between the external sources. In the present case one needs to bosonize the gluinos which transform in the  $SU(N_c)$  adjoint representation. The bosonized version of adjoint fermions can be expressed in terms of a  $SU(N_c)$   $WZW$  model of level  $N_c$  [13]. The form of the Yukawa term follows from the identification of  $:h - h^{-1}:$  with  $:\bar{\lambda} \gamma_5 \lambda:$ . The full bosonized action in the  $A_- = 0$  gauge takes the following form

$$\begin{aligned} S_{bosonized} &= S_{(N_c)}^{WZW} \\ &+ \int d^2x \operatorname{tr} \left( \frac{1}{2} (\partial_- A_+)^2 + \frac{1}{2} (D_\mu \phi)^2 + i\tilde{m}e\phi : (h - h^{-1}) :_{\tilde{m}} - eA_+ (J^+ + j_{ext}^+) \right), \end{aligned} \quad (29)$$

where  $\tilde{m}$  is the scale at which the normal ordering is performed in a similar way to the one introduced in eqn. (17). Eliminating the scalar and bosonized fermion degrees of freedom from the equations of motion which

arise from (29), one finds that the effective equation of motion of static gauge field is

$$\left( \partial_1^2 - \frac{e^2 N_c}{\pi} \frac{1}{1 - (2\tilde{m}e)^2 \frac{4\pi}{N_c} \frac{1}{\partial_1^4}} \right) A_+ = -\sqrt{2}e\rho \quad (30)$$

The meaning of this equation is that the corresponding propagator of the gluon has three poles and one of them is *tachyonic*. It turns out, as will be shown below, that the source of this unwanted pole is the fact that the fermionic contribution includes quantum corrections in the form of the axial anomaly, whereas the scalar contribution is a tree level one. This follows from the fact that we write down the classical equations of motion of (29) but it is well known that the tree level bosonized action incorporates the axial anomaly.

The correct procedure for the supersymmetric case is to consider loop effects of both the fermions and the scalars on the gluon propagator.

Before we describe the calculation associated with  $SYM_2$  model, let us review some properties of the Schwinger model and two dimensional scalar electrodynamics.

The lagrangian of massless  $QED_2$  (the Schwinger model) is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\Psi} \not{D}\Psi \quad (31)$$

The resulting classical equation of motion for the gauge field is

$$\square F = e\epsilon^{\mu\nu}\partial_\mu j_\nu, \quad (32)$$

where  $F = \frac{1}{2}\epsilon^{\mu\nu}F_{\mu\nu}$  and  $j^\mu = \bar{\Psi}\gamma^\mu\Psi$ . Classically, the right hand side of the above equation vanishes because  $\partial_\mu\epsilon^{\mu\nu}j_\nu = -\partial_\mu j_5^\mu = 0$ . However, quantum mechanically the axial current is anomalous  $\partial_\mu j_5^\mu = \frac{e}{\pi}F$  and therefore the quantum mechanical form of equation (32) is

$$\left(\square + \frac{e^2}{\pi}\right)F = 0 \quad (33)$$

which leads to the screening behavior.

In scalar electrodynamics similar phenomenon occurs. The lagrangian of the theory

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + (D_\mu\phi)(D^\mu\phi)^* \quad (34)$$

yields the following classical equation of motion for the gauge field

$$\square F = e\epsilon^{\mu\nu}\partial_\mu j_\nu, \quad (35)$$

where the scalar abelian current is  $j^\mu = -i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^* - 2ieA^\mu\phi^*\phi)$ . Note that unlike in the fermionic sector, the right hand side of (35) does not vanish even at the classical level since there is no conserved axial current in the scalar sector. The classical divergence of the “axial current” is

$$\partial_\mu\epsilon^{\mu\nu}j_\nu = -i\epsilon^{\mu\nu}(2\partial_\mu\phi^*\partial_\nu\phi - 2ie\partial_\mu(A_\nu\phi^*\phi)) \quad (36)$$

This relation is modified quantum mechanically similarly to the modification in the Schwinger model. The one loop vacuum polarization diagram modifies the axial current non conservation. In scalar electrodynamics the one loop vacuum polarization is given by the following two Feynman diagrams (the dashed line is the scalar field) which combine to

$$\Pi_{\mu\nu}(k^2) = (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \times \frac{e^2}{\pi} \int_0^1 dx \frac{k^2 x(x - \frac{3}{2})}{k^2 x(1-x) - m^2} \quad (37)$$

This expression suffers from severe infra-red divergences in the  $m^2 \rightarrow 0$  limit[14]. This technical obstacle can be overcome by the use of a point splitting of the vector current[15].

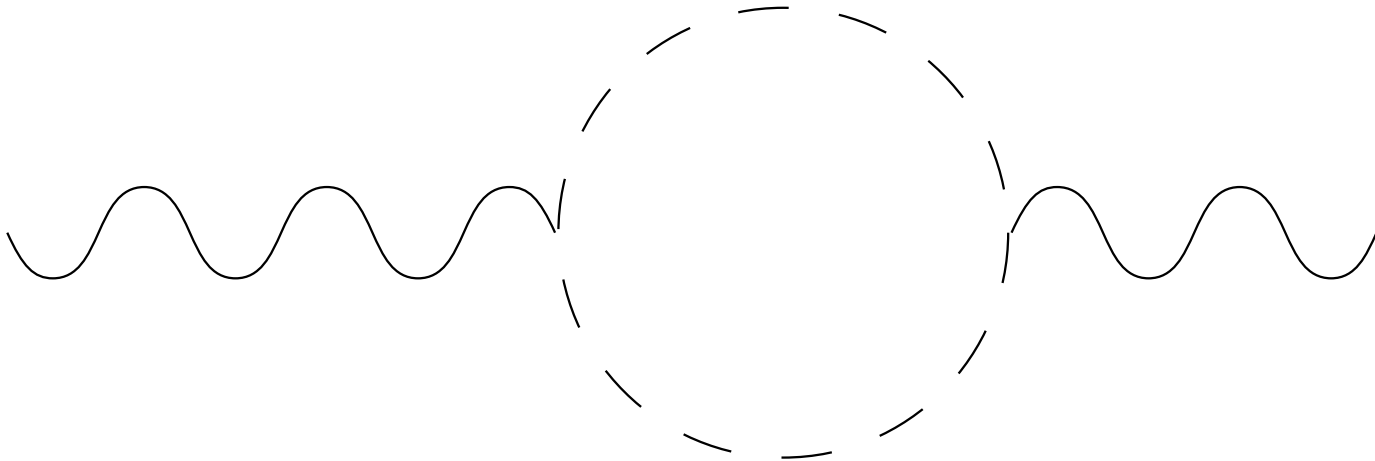
$$\begin{aligned} j^\mu = & \lim_{\epsilon \rightarrow 0} \exp(-ie \int_{x-\frac{\epsilon}{2}}^{x+\frac{\epsilon}{2}} A_\mu(y) dy^\mu) \times \\ & -i \left( \phi^*(x + \frac{\epsilon}{2}) \partial^\mu \phi(x - \frac{\epsilon}{2}) - \phi(x - \frac{\epsilon}{2}) \partial^\mu \phi^*(x + \frac{\epsilon}{2}) - \right. \\ & \left. ieA^\mu(x + \frac{\epsilon}{2}) \phi(x - \frac{\epsilon}{2}) \phi^*(x + \frac{\epsilon}{2}) - ieA^\mu(x - \frac{\epsilon}{2}) \phi(x - \frac{\epsilon}{2}) \phi^*(x + \frac{\epsilon}{2}) \right) \end{aligned}$$

In fact, the point splitting method enables us not only to get rid of the infra-red divergence but also to derive the quantum mechanical modified version of (36)

$$\partial_\mu\epsilon^{\mu\nu}j_\nu = -\frac{e}{2\pi}F - i\epsilon^{\mu\nu}(2\partial_\mu\phi^*\partial_\nu\phi - 2ie\partial_\mu(A_\nu\phi^*\phi)) \quad (38)$$

which leads to

$$(\square + \frac{e^2}{2\pi})F = -ie\epsilon^{\mu\nu}(2\partial_\mu\phi^*\partial_\nu\phi - 2ie\partial_\mu(A_\nu\phi^*\phi)) \quad (39)$$



Although the right hand side of the above equation does not vanish, it is clear that the nonlinear terms are interaction terms that can only modify the photon mass but can not make it massless. It is thus clear that the photon of scalar electrodynamics behaves in a similar way to the one of  $QED_2$ [16].

Returning back to the  $SYM_2$  case two additional complications are introduced. (i) The gauge interaction is a nonabelian one and (ii) The gluino interacts with the scalars via a Yukawa term. Nevertheless, we can follow the same procedure used for the abelian models and compute the mass of the gluon pole. The gluon equation of motion now reads

$$D_\mu D^\mu F = e\epsilon^{\mu\nu} D_\mu (J_\nu^\lambda + J_\nu^\phi), \quad (40)$$

where  $J_\mu^\lambda$  denotes the gluino vector current and  $J_\mu^\phi$  denotes the scalar vector



current. The equation for the divergence of the fermionic axial current.

$$\epsilon^{\mu\nu} D_\mu J_\nu^\lambda = -\frac{eN_c}{\pi} F + ie(\phi\bar{\lambda}\lambda + \bar{\lambda}\lambda\phi) \quad (41)$$

The factor  $N_c$  which appear in the anomaly term is due to the adjoint gluinos which run in the anomaly loop. A similar equation holds for the scalar current

$$\epsilon^{\mu\nu} D_\mu J_\nu^\phi = -\frac{eN_c}{2\pi} F - \epsilon^{\mu\nu} \partial_\mu (-2i[\phi, \partial_\nu \phi] + 2e[\phi, [A_\nu, \phi]]) \quad (42)$$

Thus the quantum version of equation (40) is

$$(D_\mu D^\mu + \frac{3e^2 N_c}{2\pi}) F = ie^2(\phi\bar{\lambda}\lambda + \bar{\lambda}\lambda\phi) - e\epsilon^{\mu\nu} \partial_\mu (-2i[\phi, \partial_\nu \phi] + 2e[\phi, [A_\nu, \phi]]) \quad (43)$$

which means that the gluon propagator has only a single pole which is massive. Though our results are based on one loop calculations, it seems that higher order corrections - which involve gluino and scalar interactions cannot spoil the massive nature of the gluon (but may shift its mass). The implication of the last equation on the potential between external charges is clear. The interaction mediated by the exchange of these massive modes is necessarily a screening one. The potential takes the form of eqn.(16) with a range that behaves like  $\sim [e\sqrt{N_c}]^{-1}$ .

## 6 Summary

The study of the interplay between screening and confining phases is a basic question in strongly interacting systems. Usually due to the lack of adequate perturbation expansion, exact statements about confinement versus screening cannot be made. One system that evades this fate is  $QCD_2$ . Special powerful techniques that are applicable only in 2D systems enable one to derive significant results. It is because of these results that  $QCD_2$  may serve as an important laboratory to study real life  $QCD$ .

Evidence that massless  $QCD_2$ , regardless of the quarks representations, is in a screening phase was presented in the paper of Gross et. al.[3]. The dynamical quarks were shown to screen external charges even if the latter are in a representation that cannot be composed of those of the dynamical

ones. It was further shown [3] that once a mass term is turned on the dynamical fermions develop a non-vanishing string tension namely, a confining potential. Tensionless strings occur only in the particular cases that the representation of the external quarks could be gotten by a composition of the dynamical ones. These results were argued using several different methods. In particular, the potential was extracted by substituting an abelian solution of the equations of motion of both the abelian theory and certain non-abelian ones.

In the present paper we derive additional supporting evidence for the picture drawn in [3]: (i) We justify the use of the equations of motion by applying a large  $N_f$  expansion; (ii) We extend the prove of screening mechanism by using “semi-classical” external charges; (iii) Show the confining potential for a bosonized massive model; (iv) Show that the 2D super YM theory has a screening behavior.

A given massless multi-flavor  $QCD_2$  model is a point in the two dimensional  $(N_c, N_f)$  grid. The domain of  $N_f = 1$  and large  $N_c$  was described in the seminal work of 't Hooft [1] in the form of the confining mesonic spectrum. The analysis of 't Hooft was insensitive to the question of whether the quarks are massless or massive. One may wrongly get the impression that the massless theory confines. In fact, this limit is not adequate for the study of the question of confinement versus screening. Assume that the potential is of the form of leading term of (15). Recall that in the large  $N_c$  limit,  $e^2 N_c$  is kept finite. This implies that the potential behaves like  $(1 - e^{-\frac{\tilde{\mu}}{\sqrt{N_c}} R}) \sim \frac{\tilde{\mu}}{\sqrt{N_c}} R \times (1 - \frac{1}{2} \frac{\tilde{\mu}}{\sqrt{N_c}} R + o(\frac{1}{N_c}))$  for fixed  $R$  and large  $N_c$ , with  $\tilde{\mu}^2 = \frac{e^2 N_c}{\pi}$  a finite constant. Now it is clear that in the limit of  $N_c \rightarrow \infty$  the potential looks like a linear potential, and thus one cannot discriminate between the two scenarios.

The opposite corner in the grid of theories is that of finite  $N_c$  and large  $N_f$  with  $e^2 N_f$  kept finite. It can be shown[17] that this limit corresponds to an approximate system of  $N_c^2 - 1$  abelian theories. In that case the screening nature can be attributed to an exchange of Schwinger-like massive modes. The present work, as well as [2], indicates that no “phase transition” should be expected when passing to models with small number of flavors. The screening nature of the potential may indicate that the theory with finite  $N_c$  has in its spectrum states of masses of the order of  $e$ . In the large  $N_f$ , using the analogy with the massive Schwinger model, one can get a general picture

of the passage to a confining behavior. The mass of the massive state of the Schwinger model is shifted once quark mass is turned on. But an additional light state emerges. Exchange of the latter mode causes confinement.

We would like to emphasize again, that studying the quantum system by analyzing the corresponding equations of motion is a justified approximation only provided that the classical configurations dominate the functional integral. This condition is obeyed in the large  $N_f$  limit since in that case an  $N_f$  factor that can be put in front of the whole action of the colored sector, and thus plays the role of  $\frac{1}{\hbar}$ .

Another improvement over the analysis of [3] is achieved by implementing the idea of [5, 6] to introduce the density of the external quarks in terms of non-commuting matrices. In that way the non-abelian nature of the large  $N_f$  limit of theory manifests itself in the form of non vanishing commutator terms whereas using the ansatz of [3] for the external charges has an abelian behavior in the same limit.

It may seem that the analysis of the question of screening versus confinement in  $SYM_2$  could follow very similar lines as those of  $QCD_2$ . However, it turns out that the equations of motion that follow from the action expressed in terms of bosonized gaugino fields, lead to unacceptable conclusion. In particular it reveals a tachyonic pole to the gauge field. This situation occurred due to an unbalanced treatment of the gaugino and scalar degrees of freedom. It is only after including quantum correction also to the scalar fields, in the form of point split currents, that a meaningful result could have been extracted. The latter corresponds indeed to a screening phase as was conjectured in [3].

Many open questions associated with the interplay between confinement and screening are still unresolved. One is the derivation of the string tension for massive dynamical quarks in any representation of color group and for any representation of the external quarks. It is speculated that this string tension is  $\sim \mu m$ , where  $m$  is the dynamical quark mass and  $\mu$  is proportional to the coupling constant, whenever the test charges *cannot* be composed of the dynamical charges, whereas when the test charges *can* be composed of the dynamical charges - the string tension is expected to be vanish. The supersymmetric  $YM_2$  has further extensions. In particular the large  $N_c$  limit of the  $N = 8$  supersymmetric case has an important significance in relation to the matrix model representation of M theory. Last but not least is obviously the implications to 4D  $QCD$ . In particular an interesting question is whether

the 2D large  $N_f$  expansion sheds any light on the 4D one.

## Acknowledgements

We are grateful to S. Yankielowicz for participating in the early stages of this project and for many useful conversations. We would like also to thank A. Zamolodchikov for illuminating discussions. One of us (J.S) would like to thank Y. Frishman for many fruitful discussions. One of us (A.A) would like to thank N. Itzhaki for several illuminating discussions.

## A Appendix - The $Q\bar{Q}$ Algebra

The Appendix is based on refs. [5, 6].

Regarding the components of the Gluon fields as operators, ordering ambiguities may appear in products such as  $f^{abc}\partial_1 A_+^b A_+^c$ . These ambiguities are resolved by the transcription

$$[u, v]^a = \frac{1}{2}f^{abc}(u^b v^c + v^c u^b) \quad (44)$$

where  $a, b, c, \dots = 1, 2, 3, N^2 - 1$  are  $SU(N_c)$  indices. The algebra is spanned by four operators:  $Q^a \otimes 1$ ,  $1 \otimes \bar{Q}^a$ ,  $f^{abc}Q^b \otimes \bar{Q}^c$  and  $d^{abc}Q^b \otimes \bar{Q}^c$ . Where  $Q^a$ ,  $\bar{Q}^a$  are  $SU(N_c)$  generators.

Using the following identifications

$$e_1^a = \frac{2}{N}(Q^a \otimes 1 + 1 \otimes \bar{Q}^a), \quad (45)$$

$$e_2^a = \frac{4}{N^2}(Q^a \otimes 1 - 1 \otimes \bar{Q}^a) - \frac{4}{N}d^{abc}Q^b \otimes \bar{Q}^c, \quad (46)$$

$$e_3^a = -\frac{4}{N}f^{abc}Q^b \otimes \bar{Q}^c, \quad (47)$$

$$e_4^a = \frac{(N^2 - 4)}{4N}(Q^a \otimes 1 - 1 \otimes \bar{Q}^a) + d^{abc}Q^b \otimes \bar{Q}^c \quad (48)$$

It was shown that

$$[e_i, e_j]^a = \epsilon_{ijk}e_k^a \quad i, j, k = 1, 2, 3 \quad (49)$$

$$[e_i, e_4]^a = 0 \quad (50)$$

and the set is orthogonal

$$tr \ e_i^a e_j^b = \frac{4}{N}\delta_{ij}\delta^{ab} \quad (51)$$

This means that the  $SU(N_c)$  algebra is actually reduced to a  $SU(2) \otimes U(1)$  problem.

Using the above algebra, it is easy to see that  $tr [Q, \bar{Q}]Q = tr [Q, \bar{Q}]\bar{Q} = 0$  and  $tr [Q, \bar{Q}][Q, \bar{Q}] = \frac{N^2}{16}tr \ e_3^a e_3^a = \frac{N^2}{2}\frac{N^2-1}{2N}$ .

## B Appendix - Calculation of $A^{(5)}$

This Appendix contains the calculation of  $j^{(4)}$  and  $A^{(5)}$  which are needed to determine the potential in the case of massless dynamical quarks (Section 3).

The next to next to leading order set of equations which determines  $A^{(5)}$  is

$$\begin{aligned}\frac{1}{\sqrt{2}}\partial_1 J^{(4)} + [A^{(3)}, J^{(0)}] + [A^{(1)}, J^{(2)}] &= -\frac{1}{2}\mu^2 \frac{1}{\sqrt{2}}\partial_1 A^{(5)} \\ \frac{1}{2}\partial_1^2 A^{(5)} &= -(J^{(4)} + \frac{1}{\sqrt{2}}j^{(4)}) \\ \partial_1 j^{(4)} + \frac{1}{\sqrt{2}}[A^{(3)}, \rho] + \frac{1}{\sqrt{2}}[A^{(1)}, j^{(2)}] &= 0\end{aligned}$$

After substitution of  $A^{(1)}$ ,  $A^{(3)}$  and  $J^{(0)}$ ,  $J^{(2)}$  we find

$$\begin{aligned}J^{(4)} &= -\frac{1}{2}\mu^2 A^{(5)} \\ (\partial_1^2 - \mu^2)A^{(5)} &= -\sqrt{2}j^{(4)}\end{aligned}$$

which is very similar to the leading and next to leading order sets of equations. The source  $j^{(4)}$  can be calculated by using the values of  $A^{(1)}$  and  $A^{(3)}$

$$j^{(4)} = -\frac{1}{\sqrt{2}}\frac{1}{\partial_1}[A^{(3)}, \rho] - \frac{1}{\sqrt{2}}\frac{1}{\partial_1}[A^{(1)}, j^{(2)}] \quad (52)$$

the result of the calculation is

$$\begin{aligned}j^{(4)} &= \\ &-\frac{1}{8\mu^3}e^{-2\mu R}(1 - e^{-2\mu R})\epsilon(x - R) \times [[\bar{Q}, Q], Q] \\ &-\frac{1}{8\mu^3}e^{-2\mu R}(1 - e^{-2\mu R})\epsilon(x + R) \times [[\bar{Q}, Q], \bar{Q}] \\ &+\frac{1}{8\mu^3}\left(\epsilon(x - R) - \epsilon(x + R)\right)(1 - e^{-\mu|x-R|}) + (\epsilon(x + R) + 1)(1 - e^{-2\mu R}) \times [[\bar{Q}, Q], Q] \\ &+\frac{1}{8\mu^3}\left(\epsilon(x + R) - \epsilon(x - R)\right)(1 - e^{-\mu|x+R|}) + (\epsilon(x - R) + 1)(1 - e^{-2\mu R}) \times [[\bar{Q}, Q], \bar{Q}]\end{aligned}$$

The first two lines in the expression of  $j^{(4)}$  arise from the commutator of  $A^{(3)}$  and  $\rho$ , the other lines are due to the commutator of  $A^{(1)}$  and  $j^{(2)}$ . Using  $j^{(4)}$ , a tedious calculation yields the following expression of  $A^{(5)}$

$$\begin{aligned}
A^{(5)} = & -\frac{\sqrt{2}}{8\mu^5} e^{-2\mu R} (1 - e^{-2\mu R}) \epsilon(x - R) (1 - e^{-\mu|x-R|}) \times [[\bar{Q}, Q], Q] \\
& -\frac{\sqrt{2}}{8\mu^5} e^{-2\mu R} (1 - e^{-2\mu R}) \epsilon(x + R) (1 - e^{-\mu|x+R|}) \times [[\bar{Q}, Q], \bar{Q}] \\
& -\frac{\sqrt{2}}{8\mu^5} \times \left\{ \begin{array}{ll} -2\mu R e^{\mu(x-R)} & x < -R \\ 2e^{-2\mu R} + (\mu x - \mu R - \frac{3}{2})e^{\mu(x-R)} & -R < x < R \\ -\frac{1}{2}e^{-4\mu R} e^{-\mu(x-R)} & \\ -2(1 - e^{-2\mu R}) + \frac{1}{2}(1 - e^{-4\mu R})e^{-\mu(x-R)} & x > R \end{array} \right\} \times [[\bar{Q}, Q], Q] \\
& -\frac{\sqrt{2}}{8\mu^5} \times \left\{ \begin{array}{ll} -\frac{1}{2}(1 - e^{-4\mu R})e^{\mu(x+R)} & x < -R \\ -2 + (\mu x + \mu R + \frac{3}{2})e^{-\mu(x+R)} & -R < x < R \\ +\frac{1}{2}e^{-4\mu R} e^{\mu(x+R)} & \\ -2(1 - e^{-2\mu R}) + 2\mu R e^{-\mu(x+R)} & x > R \end{array} \right\} \times [[\bar{Q}, Q], \bar{Q}]
\end{aligned}$$

## References

- [1] G. 't Hooft, “A Two-Dimensional Model for Mesons”, *Nucl. Phys.* **B75** (1974) 461.
- [2] Y. Frishman and J. Sonnenschein, “Bosonization of Colored Flavored Fermions and QCD in Two-Dimensions”, *Nucl. Phys.* **B294** (1987) 801 ; “Bosonization and QCD in Two-Dimensions”, *Physics Reports* **223** # **6** (1993) 309.
- [3] David J. Gross, Igor R. Klebanov, Andrei V. Matytsin and Andrei V. Smilga, “Screening vs. Confinement in 1+1 Dimensions”, *Nucl. Phys.* **B461** (1996) 109.
- [4] Y. Frishman and J. Sonnenschein, “ $QCD_2$ -Screening, Confinement and Novel non-abelian solutions”, hep-th 9710140.
- [5] S. L. Adler, “Classical Algebraic Chromodynamics”, *Phys. Rev.* **D17** (1978) 3212.
- [6] R. Giles and L. McLerran, “A Non-Perturbative Semi-Classical approach to the calculation of the Quark Force”, *Phys. Lett.* **79B** (1978) 447.
- [7] D. Kutasov and A. Schwimmer, “Universality in Two Dimensional Gauge Theory”, *Nucl. Phys.* **B442** (1995) 447.
- [8] R. Jackiw, L. Jacobs and C. Rebbi, “Static Yang-Mills fields with sources”, *Phys. Rev.* **D20** (1979) 474.
- [9] H. Arodz, “On Classical Yang-Mills equations with weak external sources”, *Nucl. Phys.* **B207** (1982) 288.
- [10] S. Coleman, R. Jackiw and L. Susskind, “Charge Shielding and Quark Confinement in the Massive Schwinger Model”, *Ann. Phys.* **93** (1975) 267.
- [11] E. Witten, “ $\theta$ -Vacua in Two-Dimensional Quantum Chromodynamics”, *Nuovo Cimento* **51A** (1979) 325.
- [12] S. Ferrara, “Supersymmetric Gauge Theories in two Dimensions”, *Lett. Nuovo Cim.* **13** (1975) 629.



- [13] O. Aharony, O. Ganor, J. Sonnenschein and S. Yankielowicz “*On the Twisted  $G/H$  Topological Models*”, *Nucl. Phys.* **B399** (1993) 560.
- [14] S.Coleman, “*There are no Goldstone bosons in two Dimensions*”, *Comm. Math. Phys.* **31** (1973) 259.
- [15] J. Schwinger, “*Gauge Invariance and Mass*”, *Phys. Rev.* **128** (1962) 2425.
- [16] A. Bengtsson and I. Bengtsson, “*Some properties of Supersymmetric QED in 1+1 Dimensions*”, *Nucl. Phys.* **B231** (1984) 157.
- [17] A. Armoni, J. Sonnenschein, “*Mesonic spectra of bosonized  $QCD_2$  models*”, *Nucl. Phys.* **B457** (1995) 81